GEG207 LECTURE NOTE

EE is the field of engineering that generally deals with the study of electricity, electronics and electromagnetism. Electronics Engineering is a discipline which utilizes non-linear and active components such as electron tubes and semiconductors (transistors, diode, IC) to design electronic devices and systems.

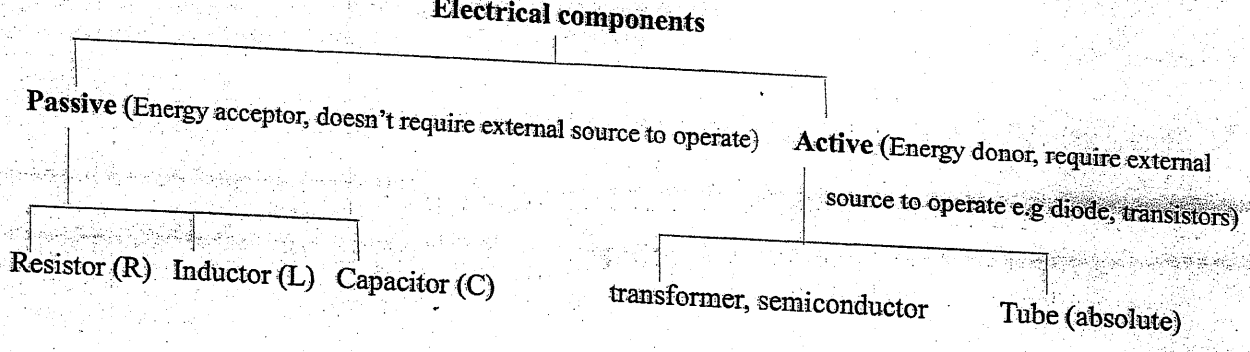
EE covers fields such as; Electronics, digital computer, power engineering, telecommunication, control system, radio frequency, signal processing, instrumentation, robotics. Prominent pioneers in electrical engineering include; Thomas Edison (Electric light bulb), Geoge Westing house (AC), Nikola Tesla (Induction motor), Guglielmo Marconi (Radio), Philo T Farnsworth (TV).

EE functions: design, develop, test and supervise the manufacture of Electronics Equipment using basic components; Conductors, Coils, Magnets, Batteries, Switches, Capacitors, Inductors, Diodes and transistors. Therefore, requiring in-depth understanding of Electrical and Electronics theory, mathematics and material in order to design circuit, perform specific functions to meet safety, reliability and energy efficient to predict how they will behave before hardware implementation.

EE application

- Communication and Entertainment- Radio, Tv, Piano
- Defensive application- Radar
- Industrial application- Automotive control, automatic door, lighting system
- Medical science- X-rays
- Instrumentation- Equipment use for research and laboratory (cathode ray, oscilloscope, voltmeter, generator, ammeter)

Electrical and Electronics Components



Definition:

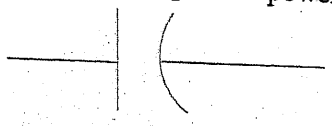
- Resistor (R) is a passive two terminal electrical component that implement electrical resistance as a circuit element. It is the relationship between V & I and measured in ohms (Ω). Resistor act to reduce current flow, at the same time act to lower voltage levels within the circuit.

  $V = IR$  R symbols

- Capacitor (C) is a passive two terminal component used to store electrical energy temporarily in an electric field. In order words it is the relationship between Q & V, measured in Farrad (F). in

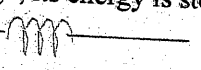
electronics it is used to block DC while allowing AC pass. In power it is used to stabilized voltage and power flow.

$Q = CV,\ (dQ = CdV$



C symbols

- Inductor (L) is a passive two terminal component which resist changes in electric current passing through it. Relationship between magnetic flux (Φ) and I, it is measured in Henry (H) $d\Phi = ldi$. Result of magnetic field around a current carrying conductor. When current flow in a conductor it creates magnetic flux. It consists of a conductor such as wire usually wounded into a coil, when current flow through, its energy is stored temporarily in a magnetic field in the coil $L = \Phi/i$

L symbol 

- Circuit: is a closed conducting path through which an electric current either flows or is intended to flow.
- Electric Network: This is a combination of various element, connected in any manner whatsoever.
- Passive Network: Is one which contains no source of emf in it. Emf; V generated by magnetic force according to faraday law.
- Active Network: Is one which contains one or more than one source of Emf.
- Parameters: The various element of an electric circuit are called its parameters like resistance, inductance, and capacitance. These parameters may be lumped or distributed
- Linear circuit: A linear circuit is one whose parameters are constant i.e they do not change with voltage or current.
- Non-linear circuit: It is that circuit whose parameters change with voltage or current.
- Bilateral circuit: A bilateral circuit is one whose properties or characteristics are the same in either direction. The usual transmission line is bilateral, because it can be made to perform it function in both directions.
- Unilateral circuit: It is that circuit whose properties or characteristics change with the direction of its operation. A diode rectifier is a unilateral circuit because it cannot perform rectification in both directions.
- Node: Is a junction in a circuit where two or more circuit elements are connected together.
- Branch: Is that part of a network which lies between two junctions.
- Loop: It is a closed path in a circuit in which no element or node is encountered more than once.
- Mesh: It is a loop that contains no other loop within it.

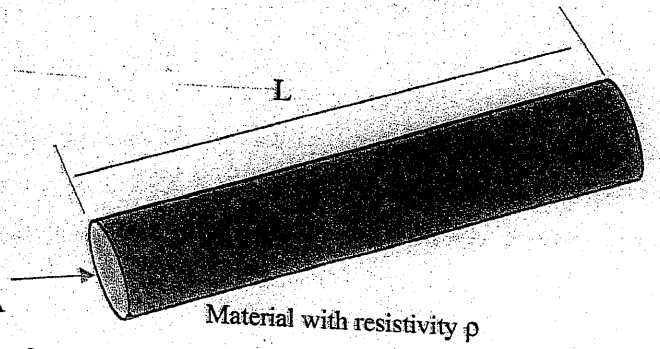
Resistance can be define as the property of a substance due to which it oppose (to restrict) the flow of electricity (ie. Electrons) through it.

$R = \frac{V}{I}$, opposition offered to a substance to the flow of I, unit is ohm (Ω).

Law of Resistance

The resistance R offered by a conductor depends on the following factors:

i. It varies directly as its length L
ii. It varies inversely as the cross-sectional A of the conductor
iii. It depends on the nature of the material.
iv. It also depends on the temperature of the conductor



$R\alpha\frac{L}{A}, or\ R = \rho\frac{L}{A}$ where A= Cross-sectional area in m^2, L= Length in meter, ρ is a constant depending on the nature of the material(Ωm)

Examples

1. A coil consists of 2000 turns of copper wire having a cross-sectional area of 0.8mm^2. The mean length per turn is 80cm and the resistivity of copper is 0.02μΩm. Find the resistance of the coil and power absorbed by the coil when connected across 110V dc supply.

   Solution.

   Length of the coil $L = mean\ length \times number\ of\ turns\ of\ the\ coil = 0.8 \times 2000 = 1600m,$

   $$A = 0.8mm^2 = 0.8 \times 10^{-6}$$

   $$R = \rho\frac{L}{A} = 0.02 \times 10^{-6} \times \frac{1600}{0.8 \times 10^{-6}}$$

   $$= 40\Omega$$

   $$Power\ absorbed = \frac{V^2}{R} = \frac{110^2}{40} = 302.5w$$

Exercises

1. Calculate the resistance of 100m length of a wire having a uniform cross-sectional area of $0.1mm^2$ if the wire is made of manganin having a resistivity of $50 \times 10^{-8}\Omega m$. If the wire is drawn out to three times it original length, by how many times would you expect its resistance to be increased? Ans= 500Ω, 9 times
2. The resistance of a conductor $1mm^2$ in cross-section and 20m long is 0.346Ω. Determine the specific resistance of the conducting material. $Ans = 1.73 \times 10^{-8}\Omega m$

Conductance and Conductivity

Conductance (G) is reciprocal of resistance. Whereas resistance of a conductor measures the opposition which it offers to the flow of current, the conductance measures the inducement which it offer to its flow.

$R = \rho\frac{l}{A}$ $or\ G = \frac{1}{\rho} \times \frac{A}{l} = \frac{\sigma A}{l}$ where σ is the conductivity or specific conductance of a conductor

$$G = {}^{1}/_{R}\ mho$$

Unit of conductance is siemens (s)

Unit of conductivity is Siemens/meter

A large conductance indicates that a material is able to conduct current well, whereas a low value of conductance indicates that a material does not readily permit the flow of charges.

Color Coding

A process called color coding is used to determine the value of resistance for a resistor, just as shown in the figure 1. below. A resistor is coated with four color band where each color determines a particular value.

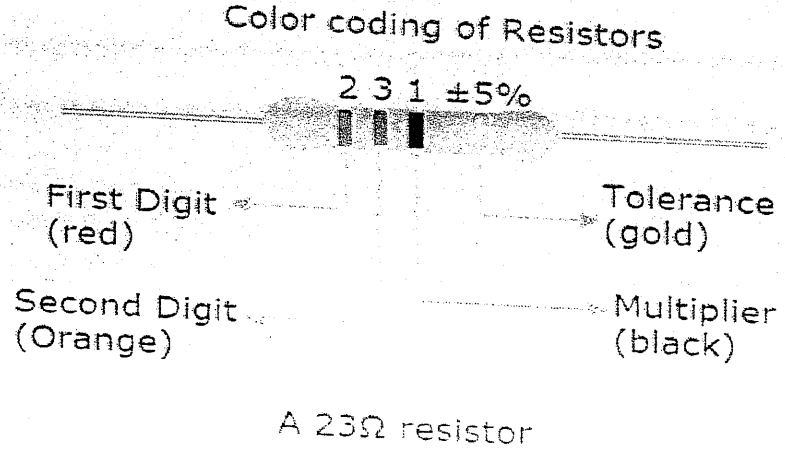


Figure 1.

The below table shows a list of values which each color indicates.

Colour	Digit	Multiplier	Tolerance
Black	0	$10^0 = 1$	
Brown	1	$10^1 = 10$	1
Red	2	$10^2 = 100$	2
Orange	3	$10^3 = 1000$	
Yellow	4	$10^4 = 10000$	
Green	5	$10^5 = 100000$	0.5
Blue	6	$10^6 = 1000000$	0.25
Violet	7	$10^7 = 10000000$	0.1
Gray	8	$10^8 = 100000000$	
White	9	$10^9 = 1000000000$	
Gold		$10^{-1} = 0.1$	5
Silver		$10^{-2} = 0.01$	10
(None)			20

The first two colored bands indicate the first and second digit of the value and the third color band represents the multiplier (number of zeroes added). The fourth color band indicates the tolerance value. Tolerance is the range of value up to which a resistor can withstand without getting destroyed. This is an important factor.

The five color band resistors are manufactured with tolerance of 2% and 1% and also for other high accuracy resistors. In these five band resistors, the first three bands represent digits, fourth one indicates multiplier and the fifth represents tolerance.

Example 1. Determine the value of a resistor with a color code Yellow, Blue, Orange and Silver.

Solution

The value of yellow is 4, blue is 6, orange is 3 which represents multiplier, silver is ±10% which is the tolerance value.

Hence the value of the resistor is $46\times10^3 = 46k\Omega$

The maximum resistance value for this resistor is $46k\Omega + 10\% = 46000 + 4600 = 50600\Omega = 50.6k\Omega$

The minimum resistance value for this resistor is $46k\Omega - 10\% = 46000 - 4600 = 41400\Omega = 41.4k\Omega$

Example 2. Determine the color code of a resistor with $4.7k\Omega \pm 5\%$

Solution.

DC NETWORK THEOREM

Ohm's law state that the voltage V across a resistor is directly proportional to current I flowing through the resistor. $V \alpha I$, $V = IR$

we can deduce to $R = \frac{V}{I}$ when temperature remain constant

Resistor in Series

Resistor are said to be connected in series if

i) The current flowing in the circuit is equal

ii) Voltage drop across each is different due to different resistance and is given by Ohm's law

iii) Sum of the voltages drops is equal to the voltage applied across the resistors.

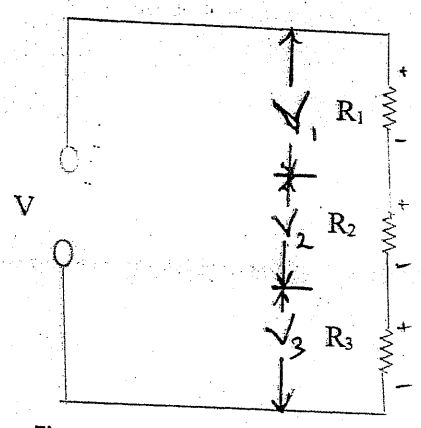


Figure 2.

Ohm's law $V = V_1 + V_2 + V_3$

$= IR_1 + IR_2 + IR_3$

$= I(R_1 + R_2 + R_3)$

$\frac{V}{I} = R_1 + R_2 + R_3$

Therefore, the equivalent resistance in series is $R_{eq} = R_1 + R_2 + R_3$

Resistance in Parallel

Resistors are said to be connected in parallel if

i) Pd across all resistors is the same
ii) Current in each resistor is different and is given by Ohm's law
iii) Total current I is the sum of the separate current through each resistor

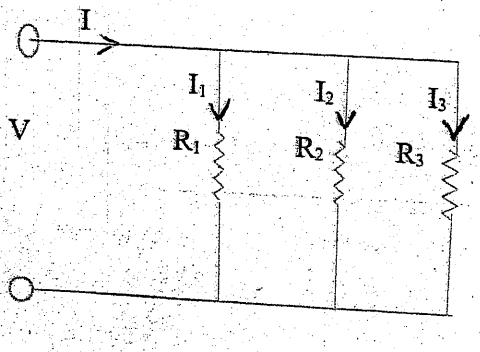


Figure 3.

From figure 3. $V = V_1 = V_2 = V_3$, and $I = I_1 + I_2 + I_3$

Therefore, $$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$I = V(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3})$$

The equivalent resistor in parallel is given as $R_{eq} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

Voltage divider rule

Consider figure 2, V = 24V, $R_1 = 2\Omega$, $R_2 = 4\Omega$ and $R_3 = 6\Omega$

To calculate the voltage drops across each resistor, we find the total resistor in the circuit which is as follows;

$$R_T = R_1 + R_2 + R_3 = 2 + 4 + 6 = 12\Omega$$

$$V_1 = V \times \frac{R_1}{R_T} = 24 \times \frac{2}{12} = 4v$$

$$V_2 = V \times \frac{R_2}{R_T} = 24 \times \frac{4}{12} = 8v$$

$$V_3 = V \times \frac{R_3}{R_T} = 24 \times \frac{6}{12} = 12v$$

Therefore, voltage across each resistor connected in series can be calculated as

$V_n = V_s \times \frac{R_n}{R_T}$ where n = 1,2,3 ....., V_s = Voltage source

Current divider rule

Consider figure 4 below,

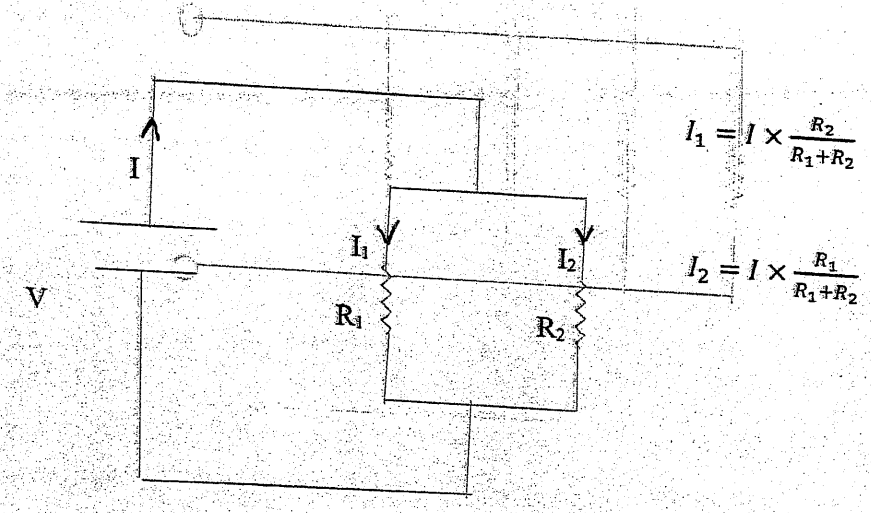


Figure 4

Example: Find the current I_1, I_2, I_3, & I_4 from the circuit in figure 5

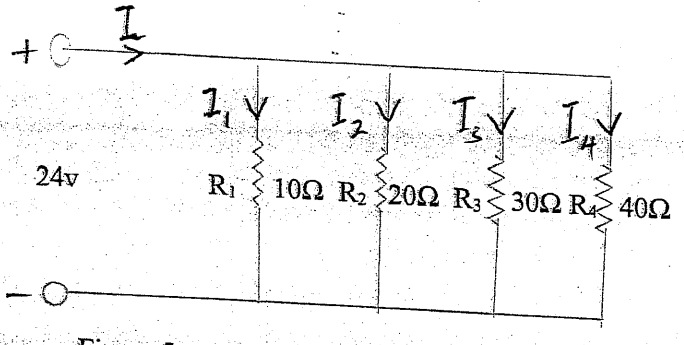


Figure 5

Solution

$$I_1 = \frac{V_S}{R_1} = \frac{24}{10} = 2.4A, \quad I_2 = \frac{V_S}{R_2} = \frac{24}{20} = 1.2A,$$

$$I_3 = \frac{V_S}{R_3} = \frac{24}{30} = 0.8A, \quad I_4 = \frac{V_S}{R_4} = \frac{24}{40} = 0.6A$$

$$I_T = I_1 + I_2 + I_3 + I_4 = 2.4 + 1.2 + 0.8 + 0.6 = 5A$$

We can verify our answer by calculating the equivalent resistance in the circuit and then apply Ohm's law,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{10} + \frac{1}{20} + \frac{1}{30} + \frac{1}{40} = \frac{12+6+4+3}{120} = \frac{25}{120}$$

$$R_T = \frac{120}{25} = 4.8\Omega$$

$$I = \frac{V}{R_T} = \frac{24}{4.8} = 5A$$

Examples

1. Find the total resistance in each of the following circuit all resistors are in ohms

a)

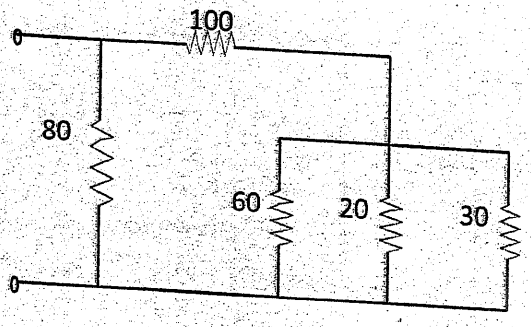


fig (a)

b)

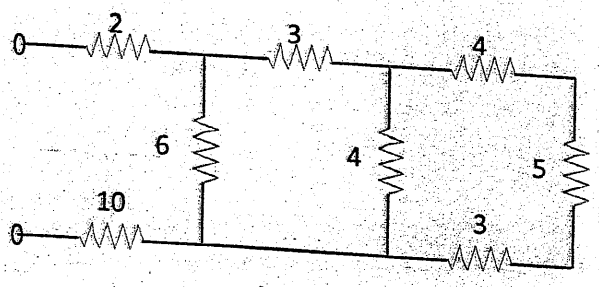


fig b)

2. Find i_o and v_o in the ckt shown in figure c. Calculate the power dissipated in the 3Ω resistor

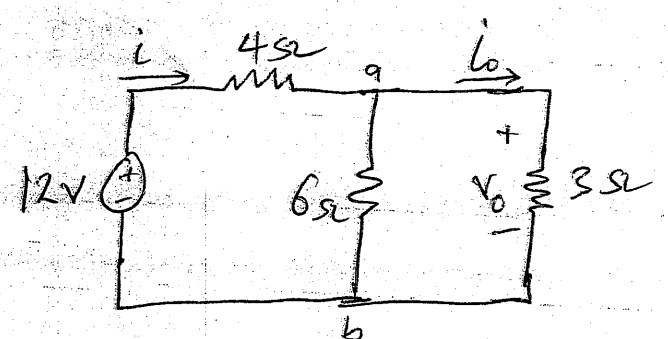
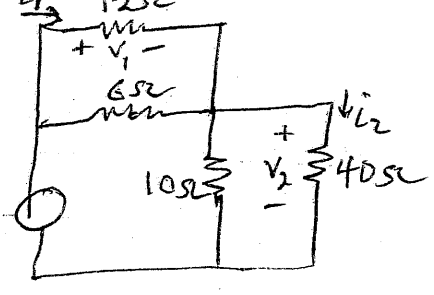


fig c

Ans: $i = 2$, $V_o = 4V$, $i_o = 4/3 A$

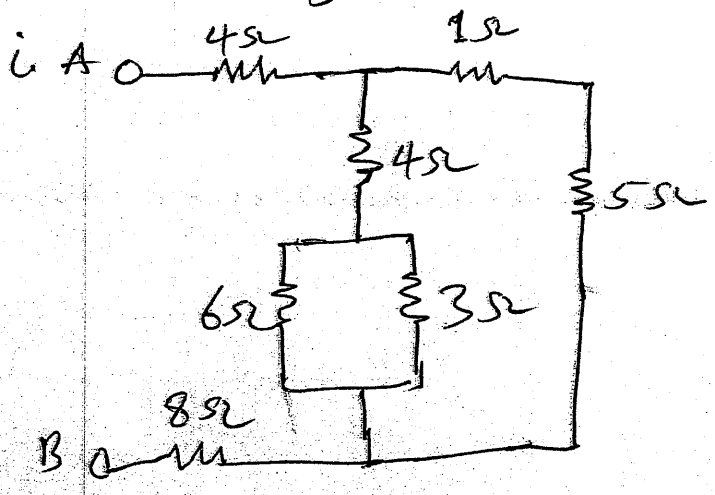
$P = 5.333W$

3. Find V_1 and V_2 in the circuit shown in fig d. Also calculate i_1 and i_2 and the power dissipated in the 12Ω and 40Ω resistors: Ans: $V_1 = 5V$, $i_1 = 416.7mA$, $P_1 = 2.083W$, $V_2 = 10V$ $i_2 = 250mA$, $P_2 = 2.5W$



Using series and parallel combination laws find the resistance between terminals A and B in the figures below.

i)



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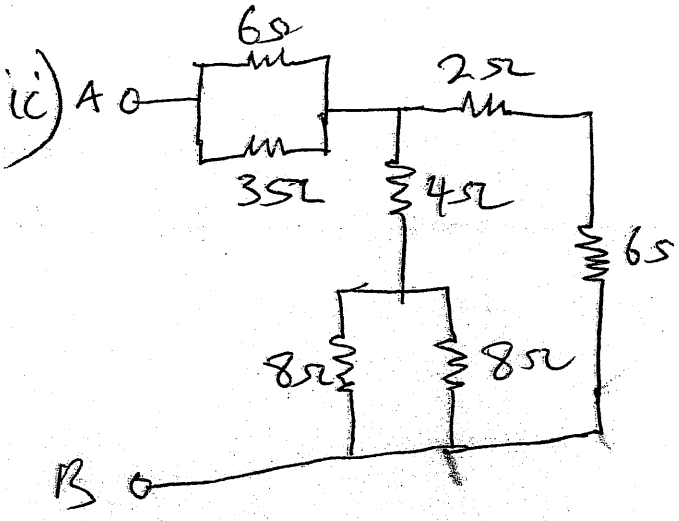

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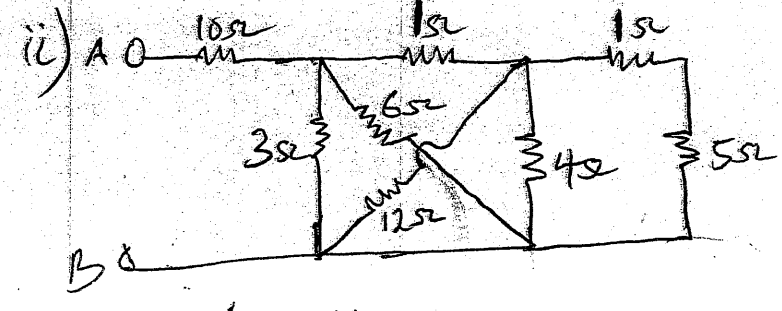
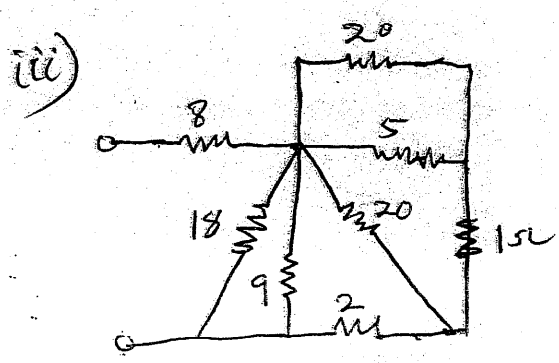


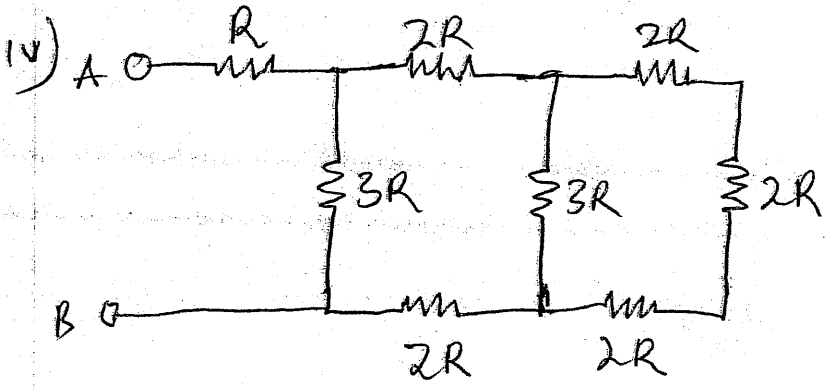
Ans = 11.2Ω

iii)



Ans: 11Ω

iv)



Short and Open Circuits

When two points of circuit are connected together by a thick metallic wire fig. 7, they are said to be short-circuited. Since "Short" has practically zero resistance, it gives rise to two important facts;

i) no voltage can exist across it because $V = IR = I \times 0 = 0$
ii) current through it (called short-circuit current) is very large (theoretically, infinity)

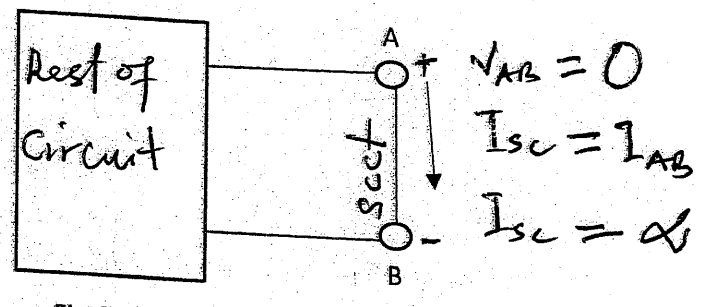


Fig. 7

Two points are said to be open-circuited when there is no direct connection between them fig. 8, obviously, an "Open" represents a break in the continuity of the circuit. Due to this break;

i) Resistance between the two points is infinite
ii) There is no flow of current between the two points.

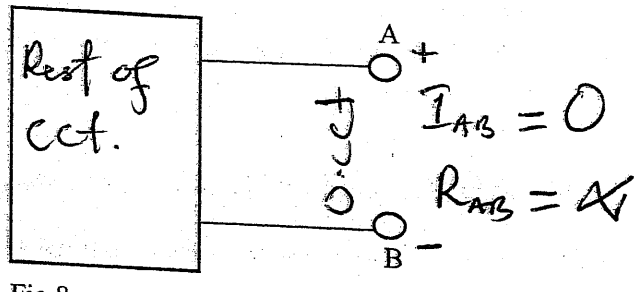


Fig.8

Short in a Series circuit

In fig 9a, $V = 12, R = R_1 + R_2 + R_3 = 6\Omega$

$$I = {}^{V}/_{R} = {}^{12}/_{6} = 2A, \quad P = I^2R = 2^2 \times 6 = 24w$$

In fig.9b 3Ω resistor has been shorted by a resistance less copper wire so that $R_{CD} = 0$

Now, total circuit resistance $R = 1 + 2 = 3\Omega$

Hence, $I = {}^{12}/_{3} = 4A$ *and* $P = 4^2 \times 3 = 48w$

In fig. 9c, 2Ω and 3Ω have been shorted out of the circuit

$R = 1,\ I = {}^{12}/_{1} = 12A,\ P = 12^2 \times 1 = 144w$

Because of the excessive current (6 times the normal value), connecting wires and other circuit components can become hot enough to ignite and burn out.

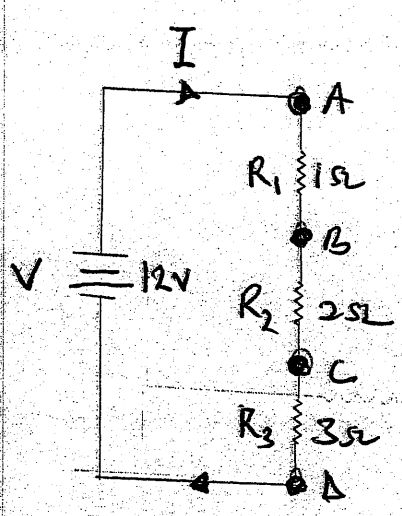


Fig.9a

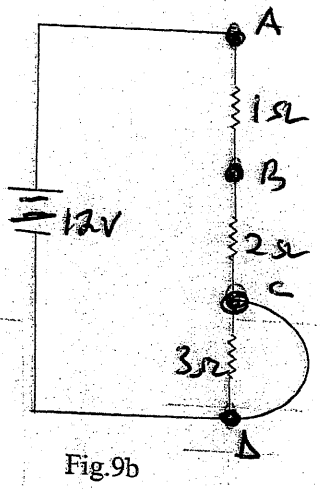


Fig.9b

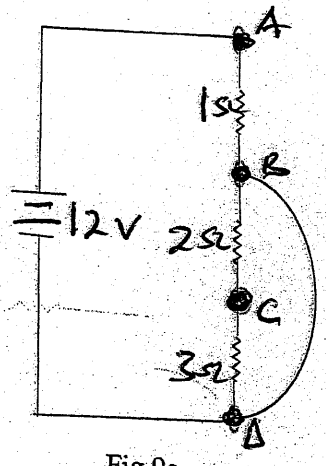


Fig.9c

Open in Series

i) Since "Open" offers infinite resistance, circuit current becomes zero. There is no voltage drop across R_1 & R_2

ii) Whole of the applied voltage (i.e 100v in this case) is felt across the open i.e across terminal A and B

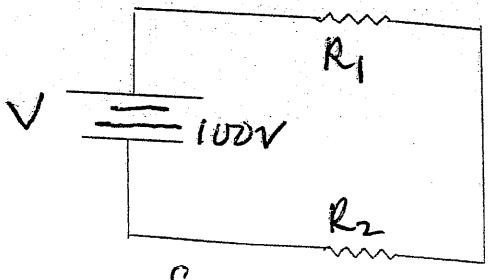


fig (a)

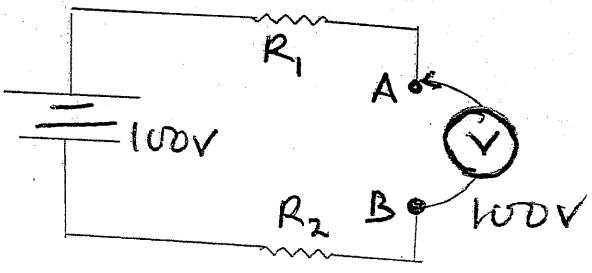
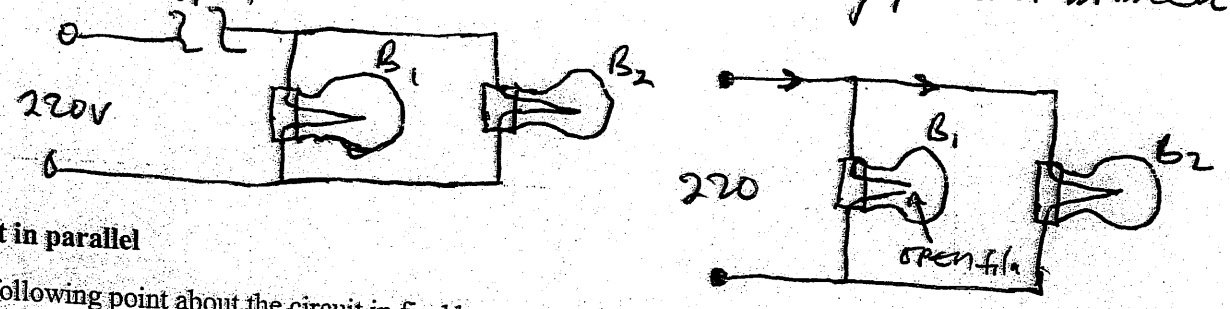


fig (b

Open in parallel

Since an 'open' offers infinite resistance, there would be no current in that part of the ckt. where it occurs. In a parallel ckt. an 'open' can occur either in the main line or in any parallel branch as shown below



Short in parallel

The following point about the circuit in fig.11a

i) Not only is R_3 short-circuited but both R_1 and R_2 are also shorted out i.e short across one branch means short across all branches.

ii) There is no current in shorted resistors. If there were three bulbs, they will not glow.

iii) The shorted components are not damaged, for example, if we had three bulbs in figure 11a they would glow again when circuit is restored to normal conditions by removing the short-circuit.

In fig11b the short-circuit across R_3 may short out R_2 but not R_1 since it is protected by R_4

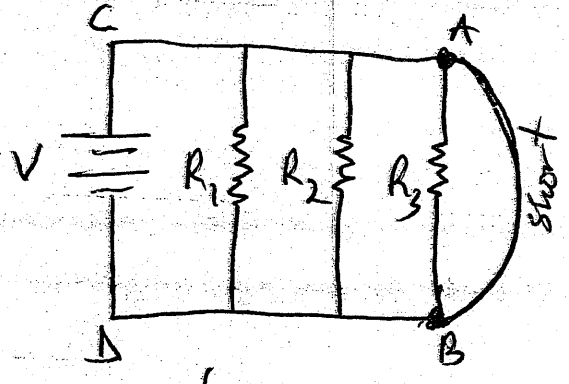


fig 11a

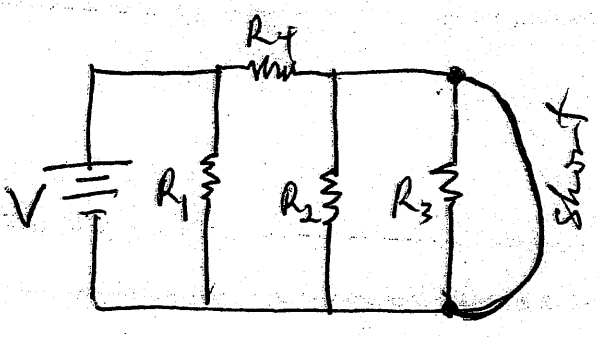
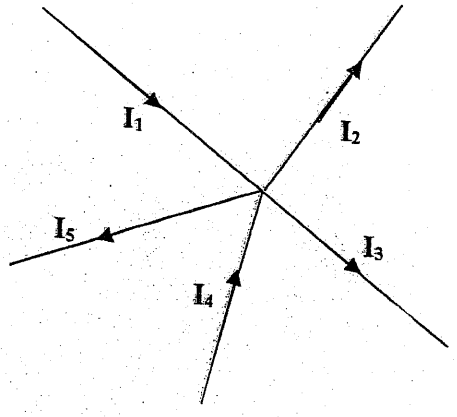


fig 11b

Kirchoff's Laws: These laws are more comprehensive than Ohm's law and are used in solving electrical networks which may not be readily solved by the latter. Kirchoff's laws, two in number, are particularly useful.

a) In determining the equivalent resistant of a complicated network of conductors and
b) For calculating the current in various conductors. The two laws are:

1. Kirchoff's point law or current law (KCL). It states as follows:
In any electrical network, the algebraic sum of the currents meeting at a point (or junction) is zero



Consider the case of a few conductors meeting at a point A in fig. above. Some conductors are having currents leading to point A, whereas as some have currents leading away from point A. assuming the incoming currents to be positive and the outgoing currents to be negative, we have

$$I_1 + (-I_2) + (-I_3) + I_4 + (-I_5) = 0$$

Or $I_1 + I_4 - I_2 - I_3 - I_5 = 0$

Or $I_1 + I_4 = I_2 + I_3 + I_5$

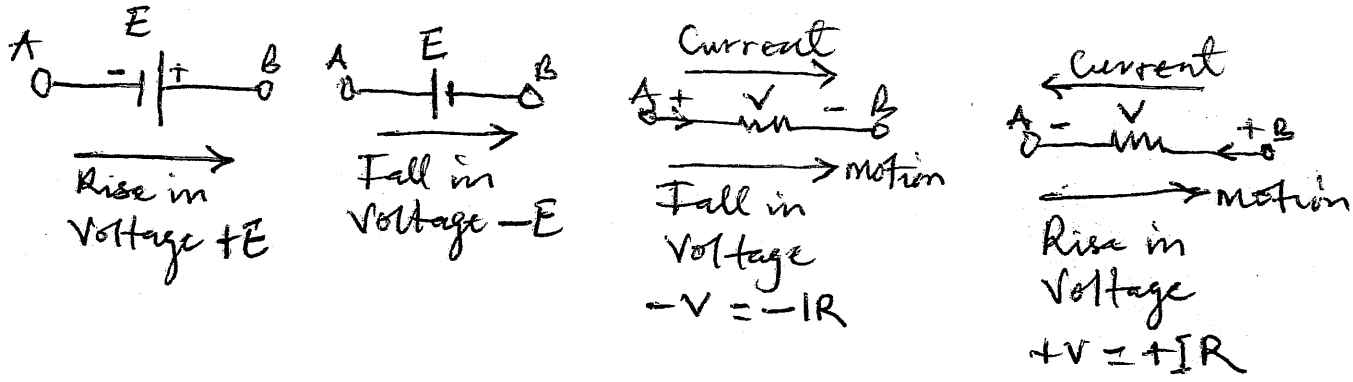
$$\sum I = 0$$

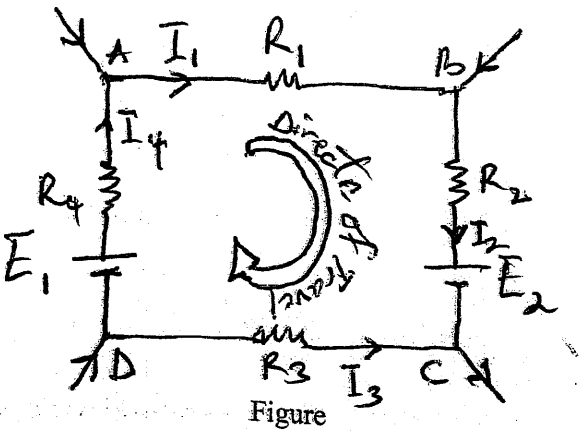
2. Kirchoff's mesh law or voltage law (KVL). It states as follows;
The algebraic sum of the products of current and resistor in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of the emfs in the path is zero.
In other words $\sum IR + \sum emf = 0$

Determination of Voltage Sign

a) Sign of a Battery Emf

Rise in voltage should be given positive sign and Fall in voltage should be given a negative sign





Figure

It is important to note that the sign of the Battery emf is independent of the direction of the current through the branch.

Sign of voltage drop:- the sign of voltage drop across a resistor depends on the direction of current through that resistor, but is independent of the polarity of any of the source of emf in the circuit under consideration.

Consider the closed path in a ABCDA in figure below. As we travel around the mesh in clockwise direction, different voltage drops will have the following sign:

I_1R_2 is -ve (fall in potential)

I_2R_2 is -ve (fall in potential)

I_3R_3 is +ve (Rise in potential)

I_4R_4 is -ve (fall in potential)

E_2 is -ve (fall in potential)

E_1 is +ve (Rise in potential)

Using Kirchoff's voltage law, we get

$$-I_1R_1 - I_2R_2 + I_3R_3 - I_4R_4 - E_2 + E_1 =$$

OR

$$I_1R_1 + I_2R_2 - I_3R_3 + I_4R_4 = E_1 - E_2$$

Examples Kirchoff's

1. Find the currents and voltages in the ckt. shown in fig. 2(a)

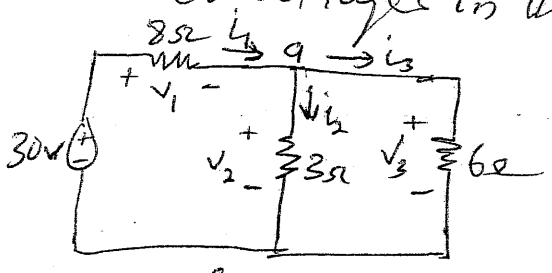


fig. 2(a)

Soln

We apply Ohm's law and Kirchoff's laws. By Ohm's law

$$V_1 = 8i_1,\quad V_2 = 3i_2,\quad V_3 = 6i_3 \qquad \text{---} (1)$$

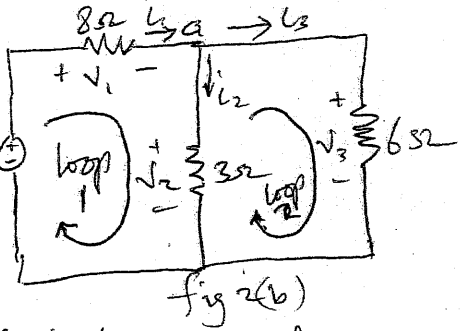


fig 2(b)

Since the voltage and current of each resistor are related by Ohm's law as shown, we are really looking for three things: (V_1, V_2, V_3) or (i_1, i_2, i_3). At node a, KCL gives

$$i_1 - i_2 - i_3 = 0 \qquad \text{---} (2)$$

Applying KVL to loop 1 as in fig 2b,

$$30 - V_1 - V_2 = 0$$

we express this in terms of i_1 and i_2 as in eqn (1) to obtain

$$30 - 8i_1 - 3i_2 = 0$$

or

$$i_1 = \frac{30 - 3i_2}{8} \qquad \text{---} (3)$$

Applying KVL to loop 2

$$V_2 - V_3 = 0 \implies V_2 = V_3 \qquad \text{---} (4)$$

as expected since the two resistors are in parallel.
or express v_1 and v_2 in terms of i_1 and i_2 eqn ④ becomes $6i_3 = 3i_2 \Rightarrow i_3 = \frac{i_2}{2}$ — ⑤

Substituting eqns ③ & ⑤ into ② ~~to obtain~~ gives

$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

$$30 - 3i_2 - 8i_2 - 4i_2 = 0$$

$$30 - 15i_2 = 0$$

$$i_2 = 30/15 = 2A$$

from eqn ⑤ $i_3 = 2/2 = 1A$

from eqn ② $i_1 + i_2 - i_3 = 0$

$$i_1 - 2 - 1 = 0$$

$$i_1 = 3A$$

Then, from eqn ①

$$v_1 = 8i_1 = 8(3) = 24V$$

$$v_2 = 3i_2 = 3(2) = 6V$$

$$v_3 = 6i_3 = 6(1) = 6V$$

2) Find the currents and voltages in the ckt. shown in figure (3)

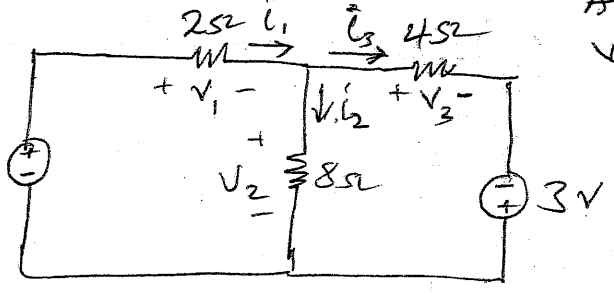


fig (3)

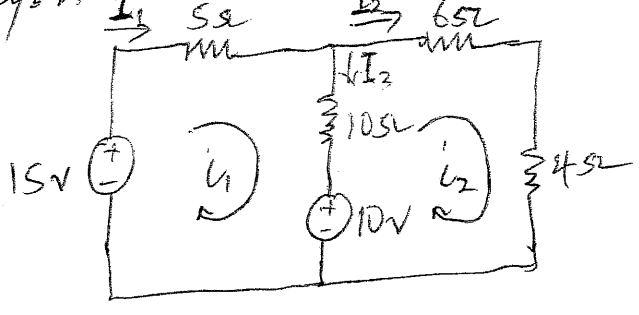
Ans. $V_1 = 3V$, $V_2 = 2V$
$V_3 = 5V$, $i_1 = 1.5A$,
$i_2 = 0.25A$, $i_3 = 1.25A$

Maxwell theorem (Mesh analysis)

Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables. Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously. Recall that a loop is a closed path with no node passed more than once. A mesh is a loop that does not contain any other loop within it. Mesh analysis applies KVL to find unknown currents.

Examples

1) Find the branch currents i_1, i_2 and i_3 using Mesh analysis



Solⁿ

From mesh 1, using KVL

$$15 - 5i_1 + 10(i_1 - i_2) - 10 = 0$$

$$5 - 5i_1 - 10i_1 + 10i_2 = 0$$

$$-15i_1 + 10i_2 = -5$$

$$3i_1 - 2i_2 = 1 \quad \text{——— (1)}$$

For mesh 2.

$$10 - 6i_2 - 4i_2 - 10(i_2 - i_1) = 0$$

$$10 - 6i_2 - 4i_2 - 10i_2 + 10i_1 = 0$$

$$10 - 20i_2 + 10i_1 = 0$$

$$10i_1 = 20i_2 - 10$$

$$i_1 = 2i_2 - 1 \quad \text{——— (2)}$$

Using Cramer's rule, we cast eqns (1) & (2) in matrix form as

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

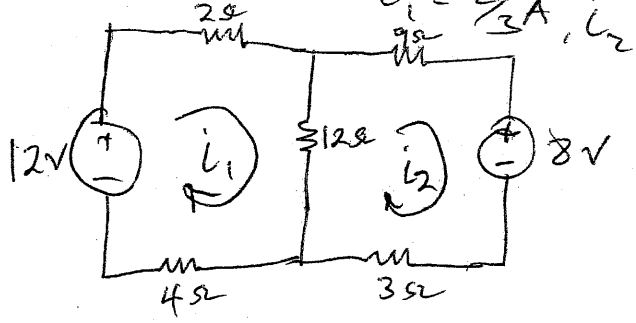
we obtain determinants

$$\Delta = \begin{vmatrix} 3 & -2 \\ 1 & 2 \end{vmatrix} = 6 - 2 = 4$$

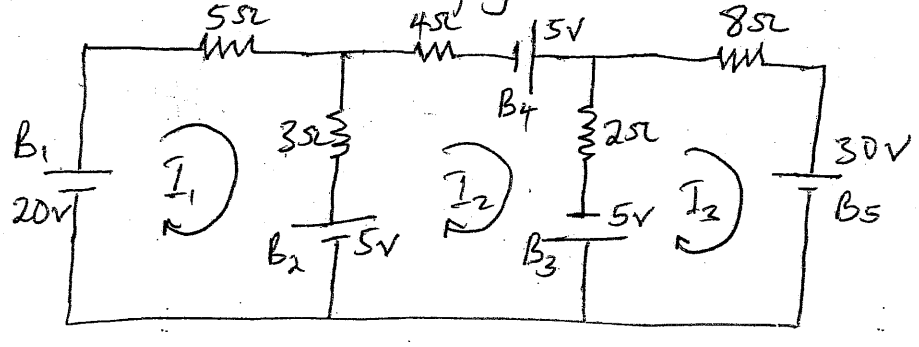
$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = 2 + 2 = 4, \quad \Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 3 + 1 = 4$$

Thus, $i_1 = \frac{\Delta_1}{\Delta} = \frac{4}{4} = 1A$, $i_2 = \frac{\Delta_2}{\Delta} = \frac{4}{4} = 1A$

(2) Calculate the mesh currents i_1 and i_2 in the cct. of figure below Ans: $i_1 = \frac{2}{3}A$, $i_2 = 0A$



3.) Determine the currents supplied by each battery in the cct shown in fig. below.



Soln

For mesh 1

$$20 - 5I_1 - 3(I_1 - I_2) - 5 = 0$$
$$20 - 5I_1 - 3I_1 + 3I_2 - 5 = 0$$
$$15 - 8I_1 + 3I_2 = 0 \Rightarrow 8I_1 - 3I_2 = 15 \quad (1)$$

Mesh 2

$$-4I_2 + 5 - 2(I_2 - I_3) + 5 + 5 - 3(I_2 - I_1) = 0$$
$$-4I_2 + 5 - 2I_2 + 2I_3 + 5 + 5 - 3I_2 + 3I_1 = 0$$
$$-9I_2 + 15 + 2I_3 + 3I_1 = 0$$
$$3I_1 - 9I_2 + 2I_3 = -15 \quad (2)$$

Mesh 3

$$-30-5-8I_3-2(I_3-I_2)=0$$

$$-35-8I_3-2I_3+2I_2=0$$

$$2I_2-10I_3=35 \quad \text{—— (3)}$$

$$\begin{bmatrix} 8 & -3 & 0 \\ 3 & -9 & 2 \\ 0 & 2 & -10 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}=\begin{bmatrix} 15 \\ -15 \\ 35 \end{bmatrix}$$

$$\Delta=\begin{vmatrix} 8 & -3 & 0 \\ 3 & -9 & 2 \\ 0 & 2 & -10 \end{vmatrix}=8(90-4)-(-3)(-30-0)+0(6-0)$$

$$=8(86)+3(-30)=688-90$$

$$=598$$

$$\Delta_1=\begin{vmatrix} 15 & -3 & 0 \\ -15 & -9 & 2 \\ 35 & 2 & -10 \end{vmatrix}=15(90-4)+3(150-70)-0$$

$$=15(86)+3(80)=1290+240$$

$$=\cancel{1050}\ 1530$$

$$\Delta_2=\begin{vmatrix} 8 & 15 & 0 \\ 3 & -15 & 2 \\ 0 & 35 & -10 \end{vmatrix}=8(150-70)-15(-30-0)-0$$

$$=8(80)+450=1,090$$

$$\Delta_3=\begin{vmatrix} 8 & -3 & 15 \\ 3 & -9 & -15 \\ 0 & 2 & 35 \end{vmatrix}=8(-315-(-30))+3(105-0)+15(6-0)$$

$$=8(-315+30)+315+90$$

$$=-2280+405=-1875$$

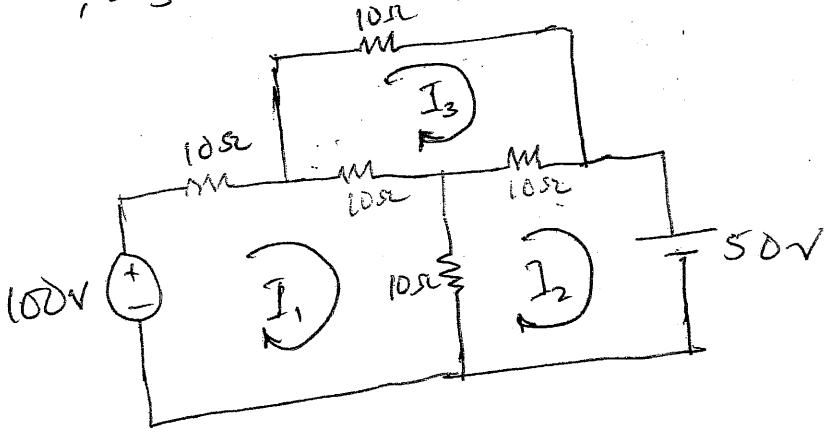
Therefore

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{1530}{598} = 2.56A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{1090}{598} = 1.82A$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-1875}{598} = -3.14A$$

4) Apply loop current method to find loop currents I_1, I_2 and I_3 in the cct. below. Ans. $I_1 = 3.75A$ $I_2 = 0$, $I_3 = 1.25A$.



NODAL ANALYSIS

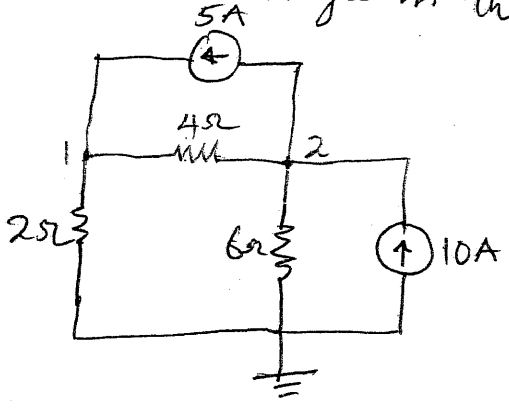
Nodal analysis provides a general procedure for analyzing cct. using node voltages as the cct. variables. Choosing node voltages instead of element voltages as cct variables is convenient and reduces the number of eqns one must solve simultaneously.

Steps to determine node voltages:

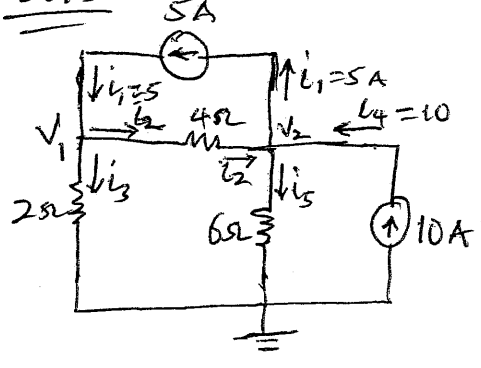
1. Select a node as the reference node. Assign voltages $V_1, V_2, \ldots V_{n-1}$ to the remaining $n-1$ nodes. The voltage are referenced with respect to the reference node.
2. Apply KCL to each of the $n-1$ ~~node~~ nonreference nodes. Use ohm's law to express the brance currents interms of node voltage.
3. Solve the resulting simultaneous eqns to obtain the Unknown node voltage.

Examples

1) Calculate the node voltages in the cct. shown below



Soln



At node 1 applying KCL and ohm's law

$$i_1 = i_2 + i_3 \Rightarrow 5 = \frac{V_1 - V_2}{4} + \frac{V_1 - 0}{2}$$

multiplying each term by 4 we obtain

$$20 = V_1 - V_2 + 2V_1$$

$$3V_1 - V_2 = 20 \quad \text{---} (1)$$

At node 2 we do the same thing and get

$$i_2 + i_4 = i_1 + i_5 \Rightarrow \frac{V_1 - V_2}{4} + 10 = 5 + \frac{V_2 - 0}{6}$$

Multiplying each term by 12 results in

$$3V_1 - 3V_2 + 120 = 60 + 2V_2$$

$$-3V_1 + 5V_2 = 60 \quad \text{---} (2)$$

Using Cramers rule

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

The determinant of the matrix is

$$\Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12$$

then

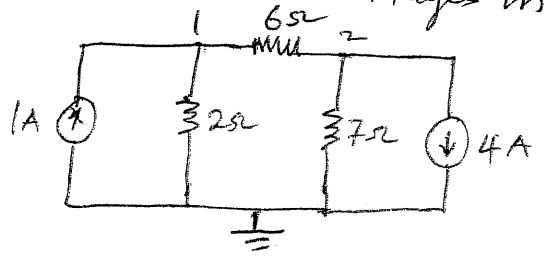
$$V_1 = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{\Delta} = \frac{100 + 60}{12} = 13.33V$$

$$V_2 = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{\Delta} = \frac{180 + 60}{12} = 20V$$

$i_1 = 5A$, $i_2 = \frac{V_2 - V_1}{4} = -1.6667A$, $i_3 = \frac{V_1}{2} = 6.666$

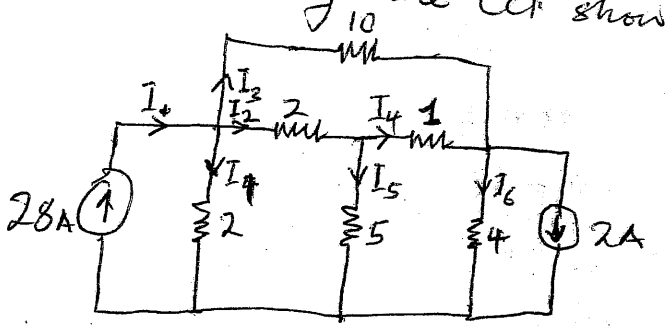
$i_4 = 10A$, $i_5 = \frac{V_2}{6} = 3.333A$.

2) Obtain the node voltages in the ccf in fig below



Ans: $V_1 = -2V$, $V_2 = -14V$

(3) Use nodal analysis method to find currents in the various resistors of the ckt shown in figure below.



Node 1

$$V_1\left(\tfrac{1}{2}+\tfrac{1}{2}+\tfrac{1}{10}\right)-\frac{V_2}{2}-\frac{V_3}{10}=28$$

or $11V_1-5V_2-V_3-280=0$ ——— (1)

Node 2

$$V_2\left(\tfrac{1}{2}+\tfrac{1}{5}+\tfrac{1}{1}\right)-\frac{V_1}{2}-\frac{V_3}{1}=0$$

or $5V_1-17V_2+10V_3=0$ ——— (2)

Node 3

$$V_3\left(\frac{1}{4}+\frac{1}{1}+\frac{1}{10}\right)-\frac{V_2}{1}-\frac{V_1}{10}=-2$$

or $V_1+10V_2-13.5V_3-20=0$ ——— (3)

The matrix form of the above three eqns:

$$\begin{bmatrix} 11 & -5 & -1 \\ 5 & -17 & 10 \\ 1 & 10 & -13.5 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}=\begin{bmatrix} 280 \\ 0 \\ 20 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 11 & -5 & -1 \\ 5 & -17 & 10 \\ 1 & 10 & -13.5 \end{vmatrix} = 1424.5 - 387.5 - 67 = 970$$

$$\Delta_1 = \begin{vmatrix} 280 & -5 & -1 \\ 0 & -17 & 10 \\ 20 & 10 & -13.5 \end{vmatrix} = 34,920$$

$$\Delta_2 = \begin{vmatrix} 11 & 280 & -1 \\ 5 & 0 & 10 \\ 1 & 20 & -13.5 \end{vmatrix} = 19,400$$

$$\Delta_3 = \begin{vmatrix} 11 & -5 & 280 \\ 5 & -17 & 0 \\ 1 & 10 & 20 \end{vmatrix} = 15,520$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{34920}{970} = 36V$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{19400}{970} = 20V$$

$$V_3 = \frac{\Delta_3}{\Delta} = \frac{15520}{970} = 16V$$

Currents

$$I_1 = \frac{V_1}{2} = \frac{36}{2} = 18A$$

$$I_2 = \frac{V_1 - V_2}{2} = \frac{36-20}{2} = 8A$$

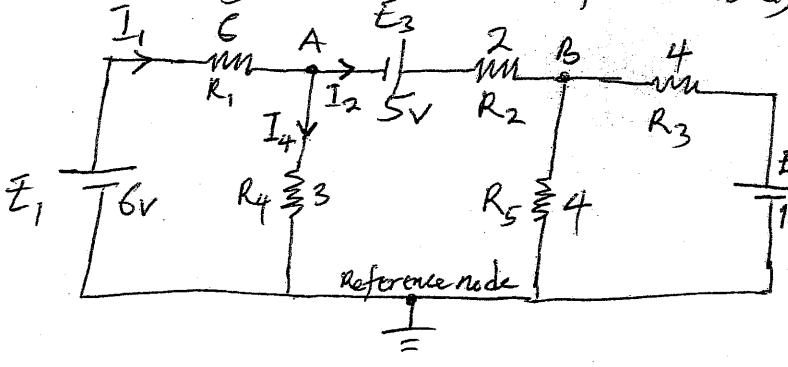
$$I_3 = \frac{V_1 - V_3}{10} = \frac{36-16}{10} = 2A$$

$$I_4 = \frac{V_2 - V_3}{1} = \frac{20-16}{1} = 4A$$

$$I_5 = \frac{V_2}{5} = \frac{20}{5} = 4A$$

$$I_6 = \frac{V_3}{4} = \frac{16}{4} = 4A$$

4) Find the branch currents in the circuit of figure below by using i) nodal analysis and ii) Mesh analysis



Ans: $V_A = 4/3$V, $V_B = 17/3$V
$I_1 = 7/9$, $I_2 = 1/3$A
$I_3 = 13/12$A
$I_4 = 4/9$A
$I_5 = 17/12$A

Superposition theorem

The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

The principle of superposition helps us to analyse a linear ckt. with more than one independent source by calculating the contribution of each independent source seperately. However to apply the superposition principle, we must keep two things in mind.

1. We consider one independent source at a time while all other independent sources are turned off. This implies that we replace every voltage source by 0V (or a short ckt.), and every current source by 0A (or an open ckt.). This way we obtain a simpler and more manageable ckt.
2. Dependant sources are left intact because they are controlled by ckt. variables.

Steps to Apply Superposition principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Examples

1. Use the superposition theorem to find v in the cct. below

Soln

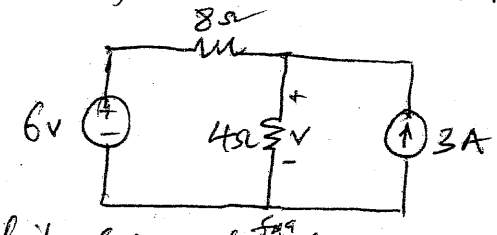


fig a

Since there are two source, let

$$V = V_1 + V_2$$

where V_1 and V_2 are the contributions due to the 6V voltage source and the 3-A current source, respectively. To obtain V_1, we set the current source to zero, as shown in the fig below. Applying KVL to the looping in fig b. gives.

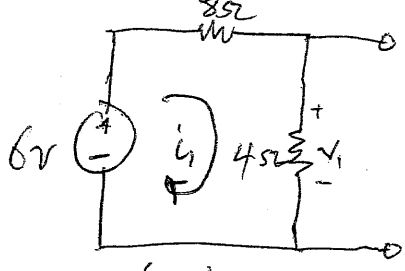


fig. b

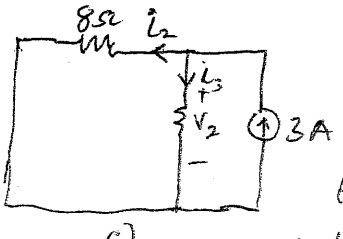
$$12i - 6 = 0 \Rightarrow i_1 = 0.5A$$

Thus: $V_1 = 4i_1 = 2V$

we may also use voltage division to get V_1 by writing

$$V_1 = \frac{4}{4+8} \times 6 = 2V$$

To get V_2, we set the voltage source to zero, as in fig c. Using current division



c)

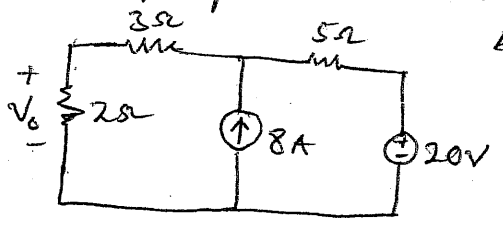
$$i_3 = \frac{8}{4+8} \times 3 = 2A$$

Hence, $V_2 = 4i_3 = 4(2) = 8V$

And we find $V = V_1 + V_2 = 2 + 8 = 10V$

2) Using the superposition theorem, find v_o in the figure below

Ans = 12V.



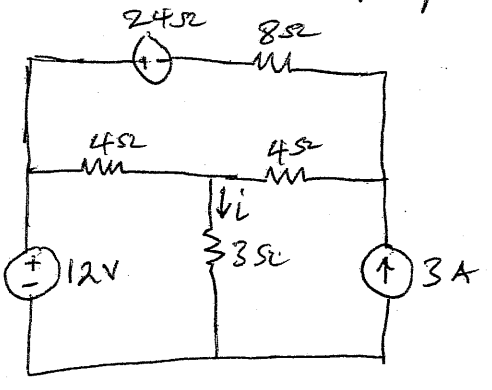
3) For the cct in fig. below use the superposition theorem to find i.

Solution

In this case we have three sources. let

$$i = i_1 + i_2 + i_3$$

where i_1, i_2, i_3 are due to 12-V, 24-V and 3A sources respectively.



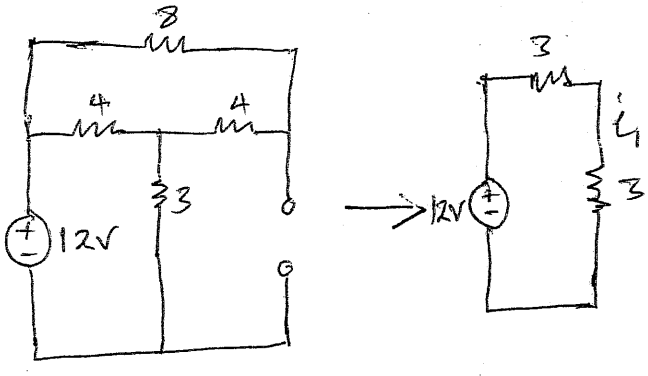
To get i_1 consider the circuit below

Combine 4Ω in series with 8Ω gives 12Ω

The 12Ω in parallel with 4Ω gives

$$12 \times 4/16 = 3\Omega$$

Thus, $i_1 = 12/6 = 2A$



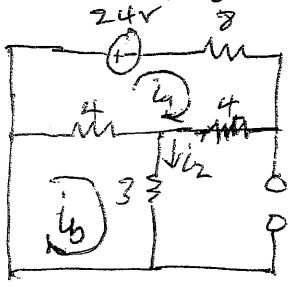
To get i_2 consider the cct. in fig. below using mesh analysis

$$16i_a - 4i_b + 24 = 0$$

or $$4i_a - i_b = -6 \quad -(1)$$

$$7i_b - 4i_a = 0$$

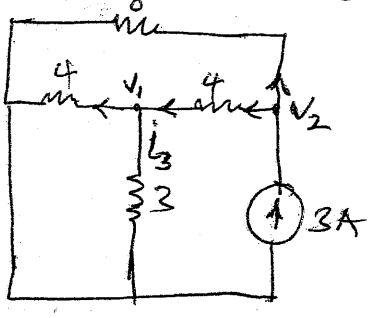
or $$i_a = \frac{7}{4} i_b \quad -(2)$$



Substituting Eq. ② into Eqn ① gives

$$i_2 = i_6 = -1$$

To get i_3, Consider the cct. in fig. below. using nodal analysis



$$3 = \frac{V_2}{8} + \frac{V_2 - V_1}{4}$$

or

$$24 = 3V_2 - 2V_1 \quad \text{—①}$$

$$\frac{V_2 - V_1}{4} = \frac{V_1}{4} + \frac{V_1}{3}$$

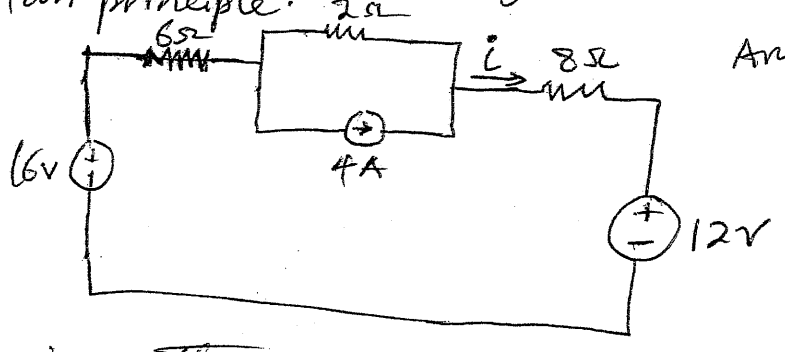
$$V_2 = \frac{10}{3} V_1 \quad \text{—②}$$

Substituting Eqn ② into Eqn ① gives $V_1 = 3$ and

$$i_3 = \frac{V_1}{3} = \frac{3}{3} = 1A$$

Thus, $i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2A$

4) Find i in the cct. in figure below. Using the superposition principle.



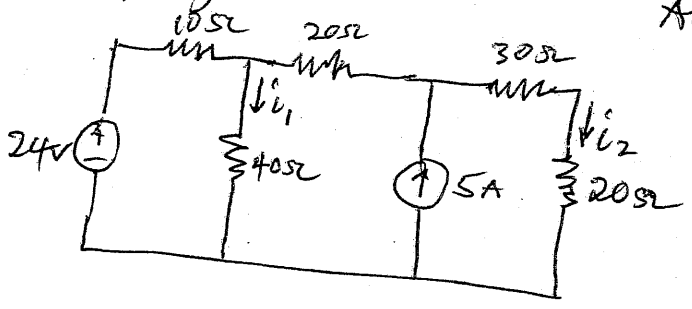
Ans: - 0.75A.

Thevenin's Theorem

States that a linear two terminal cct. can be replaced by an equivalent cct. consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open cct. voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminal when the independent sources are turned off.

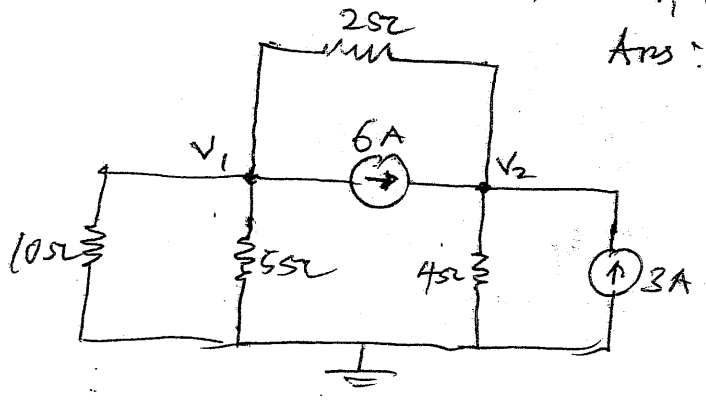
5) Use nodal analysis to find currents i_1 and i_2 in the ckt. of figure below

Ans: $i_1 = 1.072A$
$i_2 = 2.041A$.

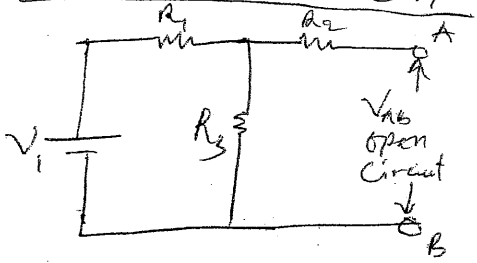


6. For the circuit below obtain V_1 and V_2

Ans: $V_1 = 4V$, $V_2 = 2V$

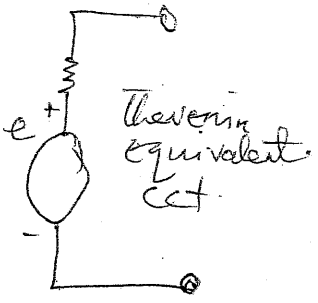


Thevenin theorem

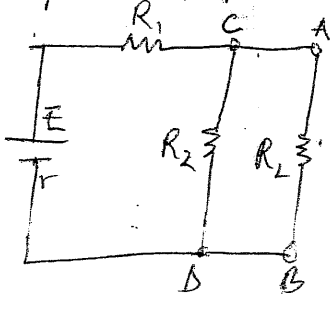


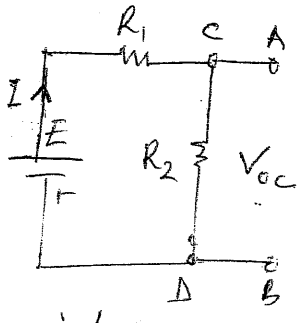
The Thevenin voltage e is the open ckt voltage at terminals A & B

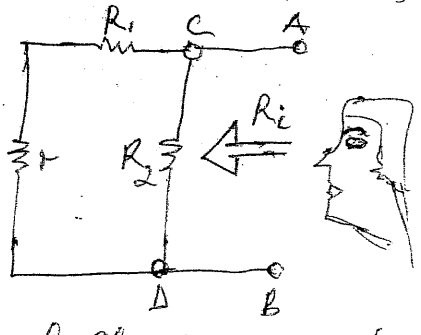
The Thevenin resistance r is the resistance seen at AB with all voltage source replaced by s.c & all current source by o.c.



It provides a mathematical technique for replacing a given network, as viewed from two output terminals, by a single voltage source with a series resistance. It makes the solution of complicated networks (particularly, electronics networks) quite quick and easy. The application of this extremely useful theorem will be explained with the help of the following simple example.







Suppose, it is required to find current flowing through the load resistance R_L as shown above. a) we will proceed as under.

1. Remove R_L from the ckt terminal A and B and redraw the ckt as shown.

2. Calc. the open circuit voltage V_{oc} which appears across terminal A and B when R_L removed.

As seen V_{oc} = drop across $R_2 = IR_2$ when I is the ckt current when A & B are open.

$$I = \frac{E}{R_1 + R_2 + r} \quad \therefore V_{oc} = IR_2 = \frac{ER_2}{R_1 + R_2 + r}$$

where r = internal resistance of the battery

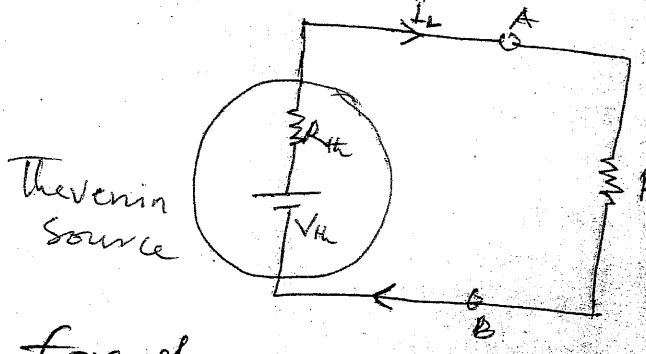
V_{oc} is called thevenin voltage V_{th}

3. Now, imagine the battery to be removed from the ckt. keeping its internal resistance r behind and redraw the ckt as shown in fig above when view inward.

$$R = R_2 // (R_1 + r) = \frac{R_2(R_1 + r)}{R_2 + (R_1 + r)}$$

This resistance is called ~~R~~ Thevenin resistance R_{th} or (R_i or R_o)

Consequently, as view from terminal A & B the whole network (excluding R_L) can be reduced to a single source called Thevenin's source whose e.m.f equals V_{oc} or V_{th} and whose internal resistance equals R_{th} or R_i as shown below

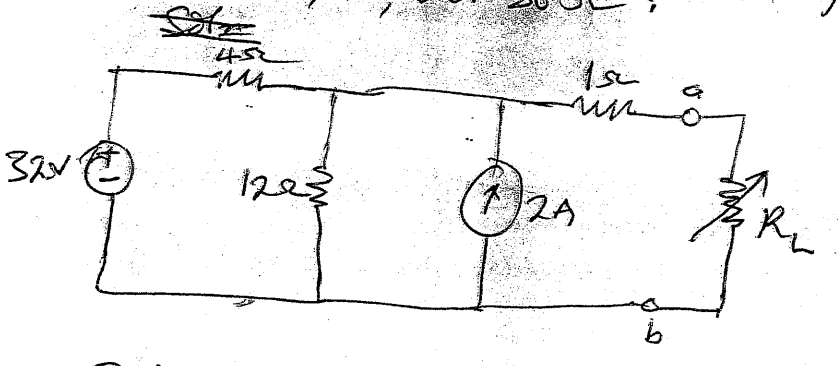


$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

$$V_L = I_L R_L = \frac{R_L}{R_{th} + R_L} \cdot V_{th}$$

Examples

1) Find the Thevenin equivalent cct. of the cct. shown below, to the left of the terminals a-b. Then find the current thru. $R_L = 6, 16,$ and $36\,\Omega$;



Soln

We find R_{th} by turning off the 32-V. voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an open cct. The cct. becomes what is shown in fig a. below.

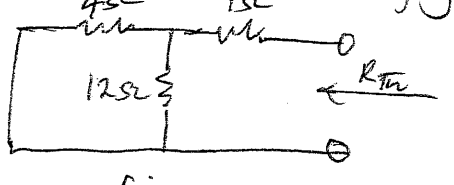


fig a.

$$R_{th} = 4 // 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\,\Omega$$

To find V_{th}, consider the ckt. in fig 6. below.

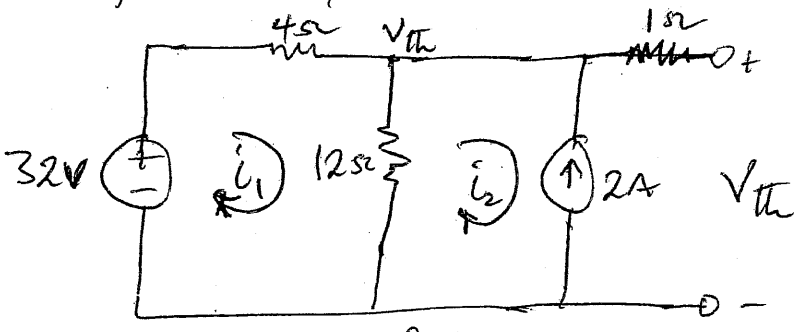


fig. 6.

Applying mesh analysis to the loops, we obtain

$$32 - 4i_1 - 12(i_1 - i_2) = 0$$

$$32 - 4i_1 - 12i_1 + 12i_2 = 0$$

$$32 - 16i_1 + 12i_2 = 0 \quad \text{and} \quad i_2 = -2A \text{ in loop 2}$$

Substitute $i_2 = -2A$ in Eqn above we get

$$32 - 16i_1 + 12(-2) = 0$$

$$32 - 16i_1 - 24 = 0$$

$$16i_1 = 32 - 24 \Rightarrow i_1 = 8/16 = 0.5A$$

Thus, $V_{th} = 12(i_1 - i_2) = 12(0.5 - (-2)) = 30V$

Alternatively we can use nodal analysis. We ignore the 1-Ω resistor since no current flow there it.

At the top node, KCL gives:

$$\frac{V_{th} - 32}{4} + \frac{V_{th}}{12} = 2$$

$$\frac{V_{th}}{4} - \frac{32}{4} + \frac{V_{th}}{12} = 2$$

$$V_{th}\left(\frac{1}{4} + \frac{1}{12}\right) - \frac{32}{4} = 2$$

$$\frac{4V_{th}}{12} - \frac{32}{4} = 2$$

$$\frac{V_{th}}{3} - \frac{32}{4} = 2$$

$$4V_{th} - 32(3) = 2(12)$$

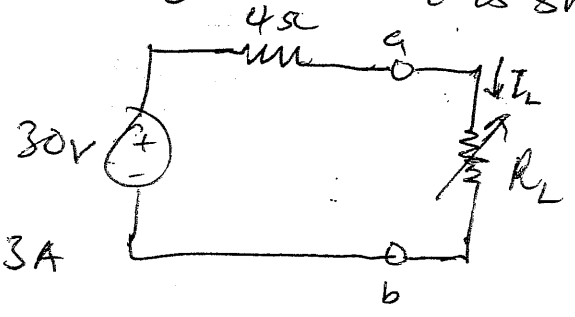
$$4V_{th} - 96 = 24$$

$$4V_{th} = 24 + 96$$

$$V_{th} = \frac{120}{4} = 30V$$

Therefore, the thevenin equivalent ckt. is shown below.

$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{30}{4 + R_L}$$



When $R_L = 6$

$$I_L = \frac{30}{4+6} = 3A$$

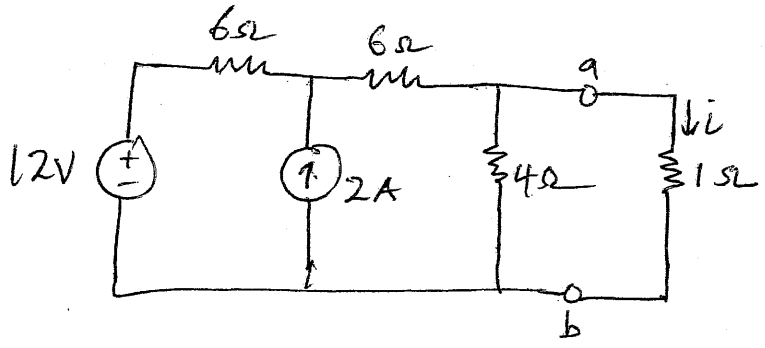
When $R_L = 16$

$$I_L = \frac{30}{4+16} = 1.5A$$

When $R_L = 36$

$$I_L = \frac{30}{4+36} = 0.75A$$

2) Using Thevenin's theorem, find the equivalent cct to the left of the terminals in the cct. in fig below. Then find i.



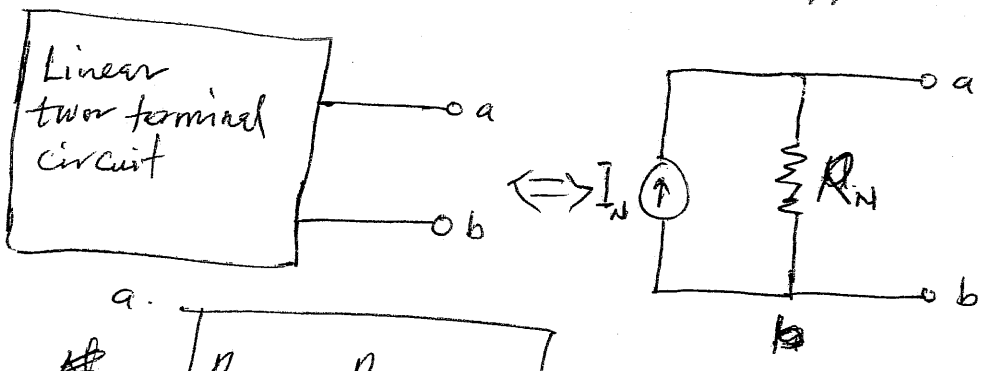
Ans: $V_{Th} = 6V$

$R_{Th} = 3\Omega$

$i = 1.5A$

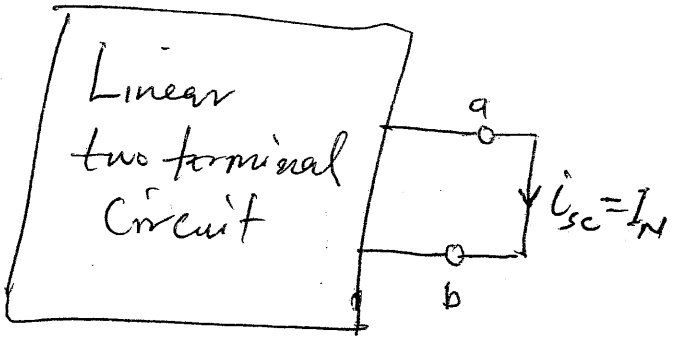
Norton's Theorem

states that a linear two terminal cct. can be replaced by an equivalent cct. consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-cct. current thru. the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.



a.

$$R_N = R_{Th}$$



$$I_N = i_{sc} \quad \text{—— (1)}$$

Observe the close relationship b/w Norton's and Thevenin's theorems: $R_N = R_{Th}$ as in previous eqn.

$$\boxed{I_N = \frac{V_{Th}}{R_{Th}}}$$

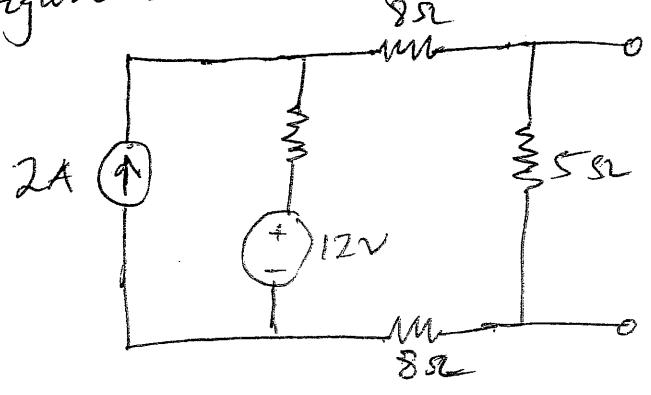
Since $V_{Th} = V_{oc}$ —— (2)

$I_N = i_{sc}$ —— (3)

$$R_{Th} = \frac{V_{oc}}{i_{sc}} = R_N \quad \text{—— (4)}$$

Examples

(1) Find the Norton equivalent cct of the cct in figure below.



Soln

We find R_N in the same way we find R_{Th} in the Thevenin cct. Set the independent sources equal to zero. This leads to the cct. in figure below.

$$R_N = 5 // (8+4+8) = 5 // 20$$

$$= \frac{20 \times 5}{25} = 4\,\Omega$$

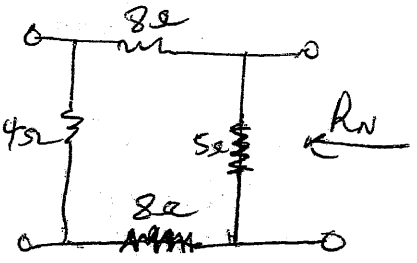


fig. a

To find I_N, we short-circuit terminals a and b, as shown in figure b. below

we ignore the 5Ω resistor b/c it has been S.C. Applying mesh analysis, we obtain.

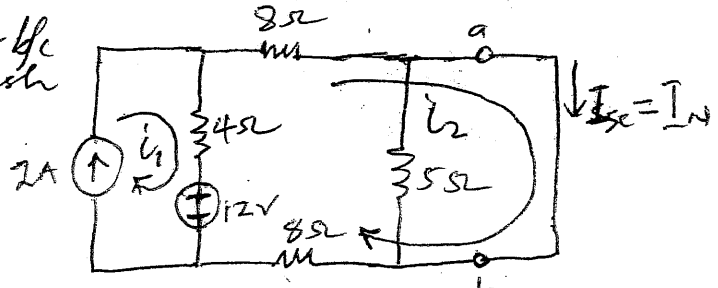


fig b

$$i_1 = 2A, \text{ and}$$

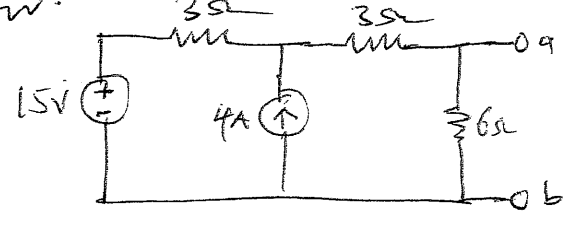
$$20i_2 - 4i_1 - 12 = 0$$

From these eqns, we obtain

$$i_2 = 1A = i_{sc} = I_N$$

Use the relationship $I_N = \frac{V_{Th}}{R_{Th}}$ to find I_N by finding V_{Th} when opening the terminal a-b.

(2) Find the Norton equivalent cct. for the cct. in fig. below.



Ans: $R_N = 3\,\Omega$

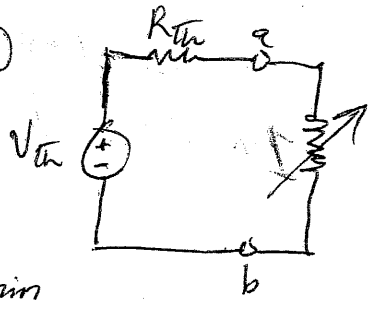
$I_N = 4.5A$.

Maximum Power Transfer

In many practical situations, a ckt is designed to provide power to a load. While for electric utilities, minimizing power losses in the process of transmission and distribution is critical for efficiency and economic reasons, there are other applications in areas such as communications where it is desirable to maximize the power delivered to a load. When we now address the problem of delivering the maximum power to a load when given a system with known internal losses. It should be noted that this will result in significant internal losses greater than or equal to the power delivered to the load.

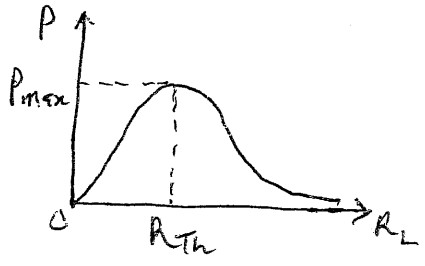
The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that we can adjust the load resistance R_L. If the entire ckt. is replaced by Thevenin equivalent except for the load, as shown in fig. below the power delivered to the load is

$$P = i^2 R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L \quad -(1)$$



Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$)

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

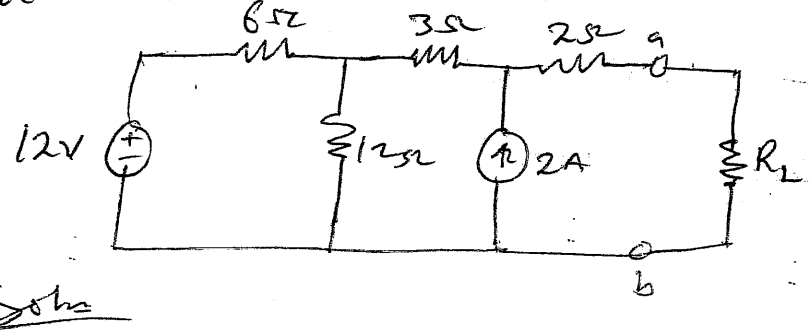


To prove the maximum power transfer theorem, we differentiate p in eqn (1) with respect to R_L and set the result equal to zero.

$$\frac{dP}{dR_L} = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L = 0 \quad \text{Prove}$$

Examples

1) Find the value of R_L for maximum power transfer in the ckt. of figure below. Find the maximum power.



Soln

We need to find the Thevenin resistance R_{Th} and the Thevenin voltage V_{Th} across the terminals a-b. To get R_{Th} we use the ckt in fig a below.

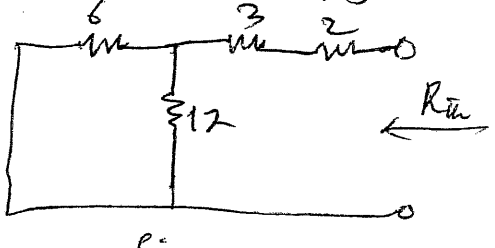


fig a.

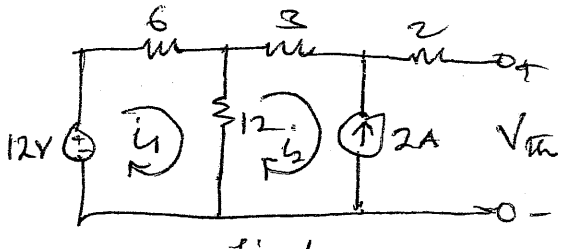


fig b

$$R_{Th} = 2 + 3 + 6/12$$

$$= 5 + \frac{6 \times 12}{18} = 9\,\Omega$$

To get V_{th}, we consider the cct in fig b on pg 25

Applying mesh analysis

$$12 - 18i_1 + 12i_2 = 0, \quad i_2 = -2A$$

Solving for i_1, we get $i_1 = -2/3$.

Applying KVL around the outer loop to get V_{th} across terminals a-b, we obtain

$$12 - 6i_1 - 3i_2 - 2(0) - V_{th} = 0$$

$$12 - 6(-2/3) - 3(-2) - V_{th} = 0$$

$$12 + 4 + 6 - V_{th} = 0$$

$$V_{th} = 22V$$

For maximum power transfer

$$R_L = R_{th} = 9\Omega$$

and the maximum power is

$$P_{max} = \frac{V_{th}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44W.$$

Exercise

(1) For the circuit in figure (a) below obtain the Thevenin equivalent as seen from Ans:- a) 3.857Ω, 4V
a) a-b b) b-c b) 3.214Ω, 15V

2) Given the circuit in figure b. below. obtain the Norton equivalent as viewed from terminal
a) a-b b) c-d. Ans:- a) 2Ω, 7A b) 1.5Ω, 12.67A

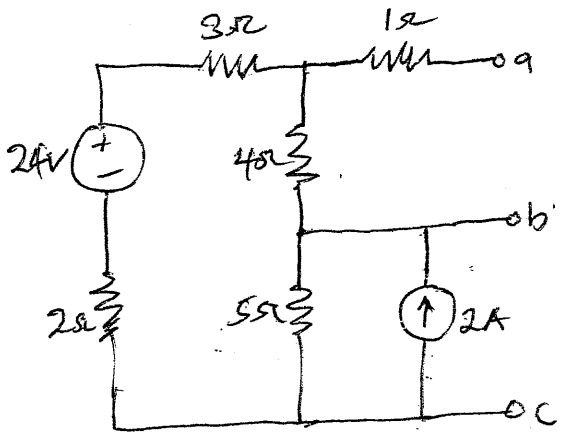


fig. (a)

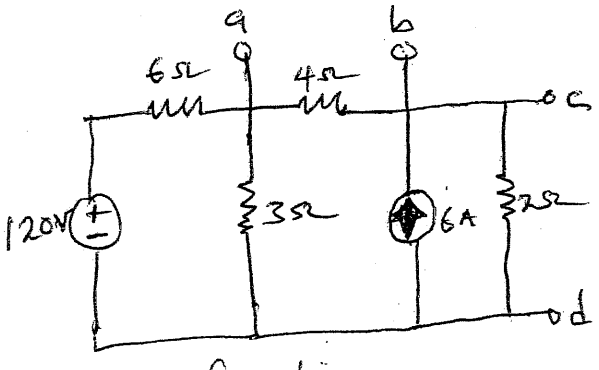
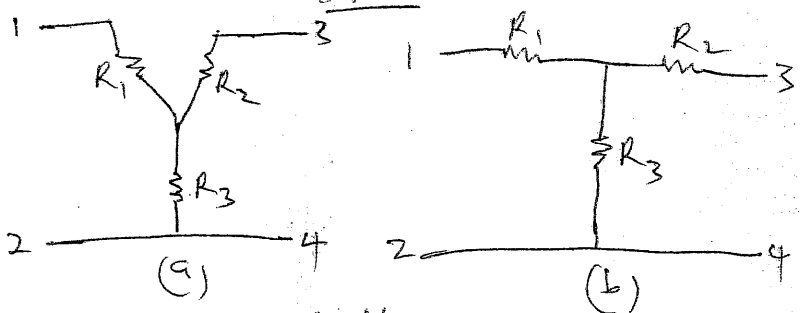


fig b.

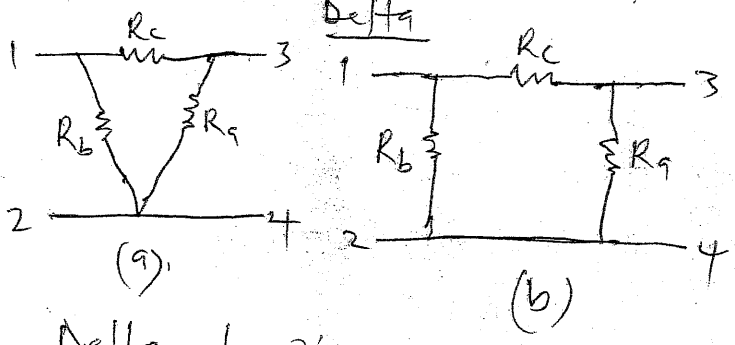
WYE/DELTA TRANSFORMATION

Star/Delta

Star



Delta



Delta to wye Conversion

Suppose it is more convenient to work with a wye network in a place where the circuit contains a delta configuration. We super impose a ~~wye~~ wye network on the existing delta network and find the equivalent resistance in the wye network. To obtain the equivalent resistances in the wye network, we compare the two networks and make sure that the resistance b/w each pair of nodes in the Δ (or π) network is the same as the resistance b/w the same pair of nodes in the Y (or T) network. For terminal 1 and 2. in figures above. For examples

$$\left.\begin{aligned} R_{12}(Y) &= R_1 + R_3, \\ R_{12}(\Delta) &= R_b // (R_a + R_c) \end{aligned}\right\} \text{———} \quad (i)$$

setting $R_{12}(Y) = R_{12}(\Delta)$ gives

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \text{———} \quad (ii)$$

according resistor in parallel for $R_{12}(\Delta)$

Similarly

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \quad \text{——— (iii)}$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \quad \text{——— (iv)}$$

Now, we substract eqn (iv) from eqn (ii)

$$(R_1 + R_3) - (R_2 + R_3) = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} - \frac{R_a(R_b + R_c)}{R_a + R_b + R_c}$$

$$R_1 + R_3 - R_2 - R_3 = \frac{R_a R_b + R_b R_c - R_a R_b - R_a R_c}{R_a + R_b + R_c}$$

$$(R_1 - R_2) = \frac{R_b R_c - R_a R_c}{R_a + R_b + R_c} \quad \text{——— (v)}$$

Then we add eqn (v) and eqn (iii)

$$(R_1 - R_2) + (R_1 + R_2) = \frac{R_b R_c - R_a R_c}{R_a + R_b + R_c} + \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$$

$$R_1 - R_2 + R_1 + R_2 = \frac{R_b R_c + R_a R_c + R_a R_c}{R_a + R_b + R_c}$$

$$2R_1 = \frac{R_a R_c + R_b R_c}{R}$$

$$R_1 - R_2 + R_1 + R_2 = \frac{R_b R_c - R_a R_c + R_a R_c + R_b R_c}{R_a + R_b + R_c}$$

$$2R_1 = \frac{2R_b R_c}{R_a + R_b + R_c}$$

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \quad \text{——— (vi)}$$

and subtracting Eqn (v) from eqn (iii) yield

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \quad \text{——— (vii)}$$

subtracting eqn (vi) from eqn (ii) we obtain

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \quad \text{——— (viii)}$$

We do not need to memorize this eqns. To transform a Δ network to Y, we create an extra node n as shown in the figure (1) below and follow this conversion rule.

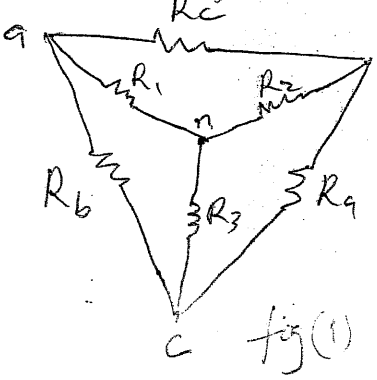


fig (1)

Each resistor in the Y network is the product of the adjacent resistors in the two adjacent Δ branches divided by the sum of the three Δ resistors.

Y to Δ Conversion

To obtain the conversion formulas for transforming Y to Δ network to an equivalent delta network, we note from eqn (vi), (viii) and (vii) that

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_aR_bR_c(R_a + R_b + R_c)}{(R_a + R_b + R_c)^2}$$

$$= \frac{R_aR_bR_c}{R_a + R_b + R_c} \quad \text{——— (ix)}$$

Dividing eqn (ix) by each of eqns (vi), (vii) and (viii) leads to the following eqns:

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \quad \text{——— (x)}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \quad \text{—(xi)}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \quad \text{—(xii)}$$

From eqns (x), (xi) and (xii) and fig (i) the conversion rule for Y to Δ is as follows.

* Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

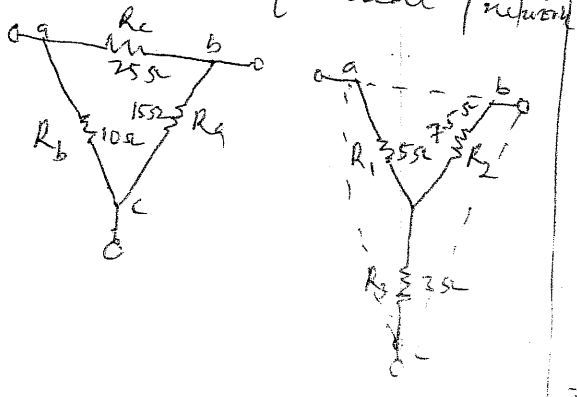
* The Y and Δ network are said to be balanced when $R_1 = R_2 = R_3 = R_Y$, $R_a = R_b = R_c = R_\Delta$ —(xiii)

Under this conditions, Conversion formulas become

$$R_Y = \frac{R_\Delta}{3} \quad \text{or} \quad R_\Delta = 3R_Y \quad \text{—(xiv)}$$

Examples

1) Convert the Δ network in fig below to an equivalent Y network



2) Transform the Y network in fig below to Δ network Ans: $R_a = 140\Omega$ $R_b = 70\Omega$ $R_c = 35\Omega$

